Effect of Model Cooling on Periodic Transonic Flow

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Experimental investigations were conducted in a small transonic intermittent tunnel on 14 and 18% thick biconvex airfoils to study the effect of model temperature on periodic transonic flows. The Reynolds number based on the model chord was in the range $0.7-0.9\times10^6$. Tests were conducted in the Mach number range of 0.7-0.85, and the model-to-tunnel stagnation temperature ratio range of 0.6-1.0. The models were cooled in liquid nitrogen to the required temperature before a run. The measurements included model surface mean pressure and temperatures and tunnel side wall dynamic pressures. The results showed a large effect of model temperature on mean pressure distribution and periodic buffet excitation levels. The effects observed are thought to correspond with what would happen to adiabatic flows at greatly increased Reynolds numbers.

Nomenclature

= pressure coefficient = pressure coefficient measured at x/c = 0.95= model chord length, mm = frequency, Hz M = freestream Mach number M_1 = Mach number just upstream of the shock = nondimensional frequency parameter, $2\pi fc/u$ p = rms pressure fluctuations, N/M^2 = kinetic pressure, N/M^2 Re = Reynolds number based on model chord = wall and adiabatic recovery temperature, K = times, s и = freestream velocity, m/s = coordinates from the leading edge of model x, y= boundary-layer thickness

Introduction

THERE are a few experiments to suggest that boundary-layer development, laminar and turbulent boundary-layer transition, and separation on a model surface are sensitive to heat transfer between the model and the flowfield. It is anticipated, from the limited data, based on mean flow measurements, that cooling $(T_w/T_{ad} < 1)$ postpones boundary-layer transition and for a turbulent boundary layer results in a flatter velocity profile, increased skin friction, and decreased boundary-layer thickness and shape factor. Cooling also reduces boundary-layer separation. For wings and bodies the combined effect due to cooling of the small increase in skin friction drag and a significant reduction in form drag (associated with the decrease in boundary-layer thickness and separation) is a reduction in total drag.

In the case of transonic flow with shock interaction, heat transfer can have a significant effect directly on the shock fect on the boundary layer approaching the shock wave.^{4,8,9} Therefore testing a model at a ratio temperature T_w/T_{ad} other than free-flight values has an effect on drag and buffet measurements in conventional and cryogenic wind tunnel and prediction of landing performance of lifting re-entry vehicles. It may be possible, as suggested by Mabey, ^{10,11} that some of the scale effects associated with Reynolds number can be identified by cooling or heating the model relative to adiabatic temperature of the air flow. Further, cooling may be an effective way of laminar flow control and reducing overall drag of an aircraft.

wave boundary-layer interaction and directly through the ef-

These observations are based on limited experimental investigations with mean flow measurements. The authors are not aware of any work on the effect of heat transfer on dynamic measurements such as buffet. This paper addresses this aspect of heat transfer and is based on experimental investigations on transonic periodic flows at low Reynolds numbers. The work reported here is restricted to laminar boundary layers. Tests with fully turbulent boundary layers will be presented in a future paper.

Review of Transonic Periodic Flows for $T_w/T_{ad} = 1.0$

Biconvex airfoils in transonic flows have an unusual form of buffet excitation, due to periodic transonic shock boundary-layer interaction. 12-17 The excitation is confined to a single frequency and occurs over a narrow range of Mach numbers (Fig. 1). Some significant features of these flows are now enumerated:

1) The periodic excitation is caused by the dynamic effect of a disturbance field interacting with a shock-induced separation.

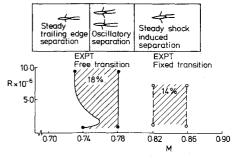


Fig. 1 Range for transonic periodic flow (after Ref. 7).

Received May 8, 1991; presented as Paper 91-1714 at the AIAA 22nd Fluid and Plasmadynamics Conference, Honolulu, HI, June 24-26, 1991; revision received Nov. 14, 1991; accepted for publication Nov. 19, 1991. Copyright © 1991 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

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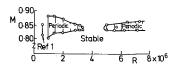


Fig. 2 Effect of Reynolds number on periodic flow range over a biconvex wing (after Ref. 5).

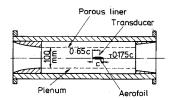


Fig. 3 Model setup in the tunnel.

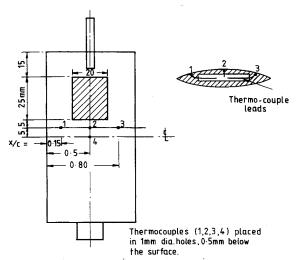


Fig. 4 Details of thermocouples.

- 2) The necessary, but not sufficient, criteria for periodic flow to occur is that the freestream Mach numbers correspond to a shock Mach number range of M_1 approximately $1.14 < M_1 < 1.24$ for low Reynolds number $(Re \le 0.6 \times 10^6)$ boundary-layer interactions and $1.22 < M_1 < 1.34$ for turbulent boundary-layer interaction at higher Reynolds numbers.
- 3) Shock waves move in antiphase on the upper and lower surfaces during shock oscillations.
- 4) The periodic flows can be modified, or even caused to disappear, if the shock interaction occurs in the vicinity of boundary-layer transition (Fig. 2). With these unusual and well-defined flow characteristics, it was expected that cooling would have a large effect on the shock oscillations with laminar, turbulent, and transitional boundary layers, and this is demonstrated in the experiments.

Experiments

Experiments were made in a 100×100 mm intermittent transonic wind tunnel with a running time of 15 s at atmospheric pressure. The tunnel had closed side walls and perforated top and bottom walls. The porosity of the perforated walls was 9.6% (Fig. 3).

The models were 14 and 18% thick biconvex airfoils of various chord length (Table 1). It was expected that the 50and 40-mm chord models would produce laminar shock wave boundary-layer interaction. The type of shock wave boundary-layer interaction was inferred from the mean pressure distributions.

The model chord sizes chosen resulted in tunnel height to model chord ratios of 2.0 and 2.5 for the 14 and 18% thick airfoils, respectively. In transonic flows, the height of the

Table 1 Test conditions $Re \times 10^6$ Airfoil, % M T_w/T_{ad} c, mm 50 0.9 0.76 - 0.880.51 - 0.990.59-0.99 40 0.70.71 - 0.83

shock wave is typically equal to chord length of the model. Therefore the tunnel interference should be small for the tunnel height to model chord ratios greater than 2. It is generally observed that the effect of tunnel interference is to lower the nondimensional frequency of shock oscillation n even in the absence of resonance, and for the tunnel height to chord ratios greater than 2, the interference effect on n is very small. Further, earlier experiments in this tunnel had shown that, for the model chord sizes chosen, there was no tunnel resonance.

The models were made of two aluminum halves with 14 static orifices (made of stainless steel tubes) on the upper surface, 5 orifices at 5-mm intervals and 9 at 2.5-mm intervals. The materials chosen for the models had low thermal expansion and construction rates at low temperatures. Compared with ambient temperature, the contraction at 198 K was less than 0.2%. Thermocouples were installed within the model, 20 close to the surface and 1 at the centerline at midspan (Fig. 4). The thermocouples were copper-constantan (type T) that had an operating temperature range of 73-673 K. The thermocouples had exposed tips to reduce the response time to approximately 70 μ s. A length of 10-15 mm of insulated leads was placed in an isothermal region and at low temperatures within the model to reduce lead wire thermal conduction errors. The halves of the models were bonded together by a resin that also had a very low expansion rate.

The model was cooled before a run in a bath of liquid nitrogen. The model was installed in the tunnel when the model temperature reached a few degrees below the required test temperature.

A Kulite pressure transducer was used for the unsteady measurements, on the tunnel side wall at x/c = 0.7 and y/c=0.17 (Fig. 3). This position can sense shock oscillations. Model cooling had no effect on the transducer sensitivity since it was found by thermocouple measurements at the transducer location that the temperature at this point changed by less than 2 K during a run.

A Scanivalve with a Druck pressure transducer (type PDCR22) and pressure storage box was used for static pressure measurements. The output from the pressure storage box was recorded on an x-y plotter. The signals from the Kulite transducer were recorded on a tape recorder (Bruel and Kjaer type 7003) for subsequent analysis by a spectral analyzer (FFT type AD3525). The thermocouple signals were logged in and analyzed by a computer (spectra-MS).

The test conditions are summarized in Table 1. Figure 5 shows typical results of T_w/T_{ad} on the model surface at three locations 1, 2, and 3 for two initial temperature settings of $T_w/T_{ad} = 0.99$ and 0.76. For the adiabatic model, the changes in T_w/T_{ad} with t are negligible. For the cooled model, the temperature of the surface increases due to heat transfer from the airflow to the model. The rate of change of temperature on the surface of the model was typically 4 K/s, which corresponds to a value of $1/T_w \delta T_w/\delta t = 2\%$. The sampling period itself was less than 1 s, during which the change in model temperatures was negligible. The change in mean static pressures on the model (as observed by continuous sampling) over a period of 1 s was negligible when compared with changes observed when the wall temperature was changed at steps of 25 K. Further, the differences in temperatures recorded by thermocouples at various locations on the model were less than 4 K. Hence the model was at nearly uniform temperature during a run and the flow could be analyzed as quasisteady. No frost formation on the model was noticed even during runs at temperatures as low as 173 K.

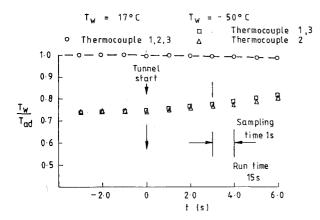
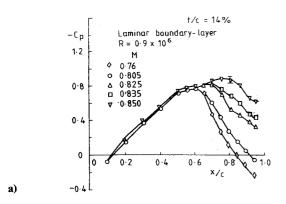


Fig. 5 Typical time histories.



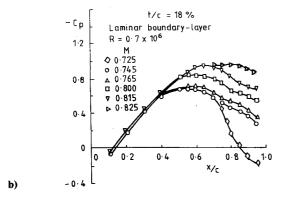


Fig. 6 Typical mean pressure distributions, $T_w/T_{ad} = 0.99$: a) 14% and b) 18%.

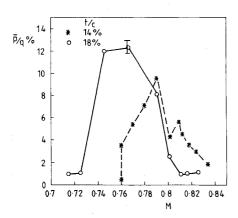


Fig. 7 Variation of unsteady pressures with Mach number $T_w/T_{ad} = 0.99$.

Results

Tests for $T_w/T_{ad}=1$

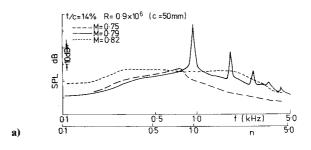
Figure 6 shows typical mean pressure distribution of the 14% thick (c=50 mm) and 18% thick (c=40 mm) airfoil at several freestream Mach numbers and at near adiabatic wall temperature conditions ($T_w/T_{ad}=0.99$). The pressure distributions are typical of laminar boundary-layer, shock wave interactions in transonic flow with an extended interaction region that increased with the increase in freestream Mach number. The pressure coefficient near the trailing edge had diverged at M=0.805 for the 14% thick airfoil, indicating trailing-edge boundary-layer separation.

Figure 7 shows a plot of airfoils of nondimensional rms dynamic pressure (\bar{p}/q) against M for both airfoils. For the 18% thick airfoil a significant increase in pressure fluctuation occurs over a narrow Mach number range 0.74–0.80, which is in general agreement with the results of McDevitt¹² and previous tests in this facility. The levels of peak pressure fluctuations, $\bar{p}/q = 13\%$, measured with the tunnel side wall pressure transducer compares with levels of 30% or more measured with a surface-mounted pressure transducer. For the 14% thick airfoil the range of Mach numbers for periodic pressure fluctuations is higher, 0.76–0.84, and peak pressure fluctuations are lower. This is a wider range for shock oscillations than in other investigations (compare Figs. 1 and 2). The second peak for the 14% thick airfoil at M=0.81 has been observed previously. The second previously.

Transonic periodic flows have a fundamental frequency and various harmonics. Figures 8a and 8b show typical spectra of pressure fluctuations for the 14 and 18% thick airfoils at three Mach numbers. The spectrum at M=0.79 has a peak at fundamental frequency of f=800 Hz, which corresponds with a nondimensional frequency n=1.00. This value compares favorably with values quoted previously. There are no peaks at M=0.75 and 0.82, indicating random rather than strongly periodic shock oscillations.

Tests for $T_w/T_{ad} < 1$

Figures 9a and 9b, respectively, show the mean pressure distributions on the 14 and 18% thick airfoils for a fixed freestream Mach number and for three temperature ratios. The Mach numbers chosen (M=0.79 for 14% and 0.765 for 18%) refer to, at adiabatic wall conditions, the presence of significant periodic oscillations and have produced relatively stronger shock wave boundary-layer interaction for the 18% thick airfoil. It is clear from these figures that cooling the



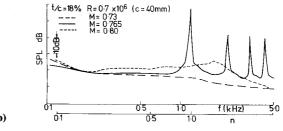


Fig. 8 Typical spectra of pressure fluctuations, $T_w/T_{ad}=0.99$: a) 14% and b) 18%.

model alters the mean pressure distribution in the region of shock wave boundary-layer interaction. Decrease in T_w/T_{ad} values has resulted in an increase in the shock Mach numbers, increase in pressure gradients in the interaction region, and a decrease in the extent of interaction.

Similar effects of cooling have been observed by Inger et al.⁸ in a transonic turbulent boundary-layer interaction and by Frishett⁶ in a supersonic turbulent boundary-layer interaction.

The effect of cooling is considerably larger with stronger shock wave boundary-layer interaction, which is in agreement with the findings of Frishett. The model proposed by Elf-strom⁷ in supersonic flow also indicates similar effects.

All of these effects are a direct result due to cooling of the reduction in boundary-layer thickness and sonic height of the boundary-layer approaching the shock wave. Cooling the model surface produces a decrease in boundary-layer thickness, flatter velocity profiles, and reduced sonic height. In transonic flow at low Reynolds numbers, the viscous layer is of the same thickness as the sonic height, ¹² and cooling has a

large effect on both dimensions. The reduction in sonic height reduces the communication across the shock wave and therefore produces an increase in pressure gradients in this region. The boundary-layer development downstream of the shock is dependent on the boundary-layer parameters at the foot of the shock, and hence cooling should have an indirect effect on the separated flow.

Figures 9a and 9b also show that in spite of the increase in shock strength associated with cooling, the pressure coefficients near the trailing edge have not changed significantly. This suggests that the reduction in T_w/T_{ad} has a favorable effect on the separated boundary layer. This must be due to a reduction in boundary-layer thickness and flatter velocity profile produced by the cooling of the boundary layer approaching the shock wave. In addition, cooling may have a direct effect on the separated boundary layer.

As seen for an airfoil at a given freestream Mach number M, cooling has resulted in a change on shock Mach number M_1 . To access the effect of cooling on trailing-edge pressures

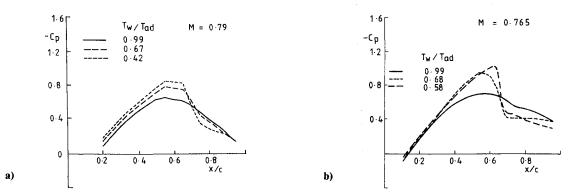


Fig. 9 Effect of wall temperature ratio on pressure distributions: a) 14% and b) 18%.

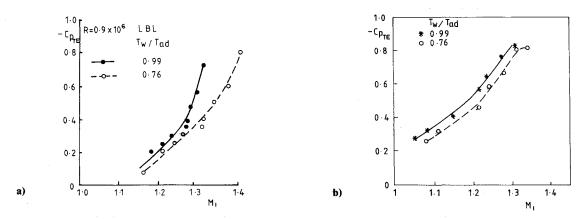


Fig. 10 Pressure coefficient near trailing edge as a function of shock Mach numbers: a) 14% and b) 18%.

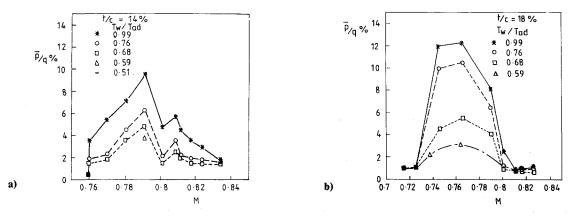
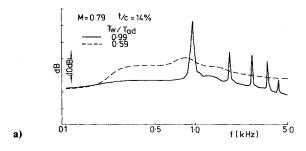


Fig. 11 Variation of wall pressure fluctuations with Mach number; several wall temperature ratios: a) 14% and b) 18%.



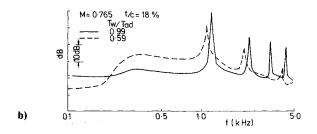


Fig. 12 Influence of T_w/T_{ad} on excitation spectra: a) 14% and b) 18%.

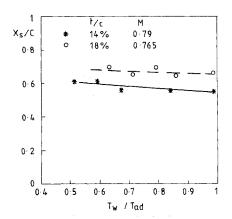


Fig. 13 Variation of shock position with wall temperature ratio.

for a given shock strength, the mean trailing-edge values of pressure coefficients are plotted against shock Mach numbers in Fig. 10. Results shown here are for two temperature ratios of 0.99 and 0.76. It can be observed that for both the 14% thick airfoil (Fig. 10a) and 18% thick airfoil (Fig. 10b), the effect of cooling for a given shock strength M_1 is to improve pressure recovery. Thus there is a large effect of cooling on the boundary-layer development downstream of the shock interaction.

Figure 11 shows the important effect of cooling on the random and periodic buffet excitation. For the 14% thick airfoil (Fig. 11a), the peak value of \bar{p}/q was reduced from 9 to 3% when T_w/T_{ad} was reduced from 0.99 to 0.51. The corresponding changes were even larger for the 18% thick airfoil (Fig. 11b). Here the peak value of \bar{p}/q was reduced from 12.5 to 3% when T_w/T_{ad} was reduced from 0.99 to 0.50. The Mach number range of shock oscillations (0.74 < M < 0.8, 0.76 < M < 0.84) for 14 and 18%, respectively, is not affected by cooling.

Figures 12a and 12b show the effect of cooling on a typical spectrum of the pressure fluctuations. Cooling reduces the levels at the fundamental frequency and the harmonics. For $T_w/T_{ad}=0.59$ the shock oscillations have virtually disappeared for the 14% thick airfoil. It is also observed that cooling has resulted in an increase in broadband fluctuation levels. These dynamic measurements suggest that cooling may stabilize boundary-layer separation and that the T_w/T_{ad} ratio is an important parameter in buffet measurements.

Figures 13 and 14 refer to typical test conditions on both airfoils at a fixed freestream Mach number and at several T_w/T_{ad} ratios in the range $0.5 < T_w/T_{ad} < 0.99$. The Mach numbers chosen were M=0.79 and 0.76 for the 14 and 18% thick airfoils, respectively. These Mach numbers give large shock oscillations.

As can be observed from Fig. 13, cooling the model had only a weak influence on the mean shock position. The shock position here was taken as the same as the position of the highest Mach number on the airfoil. However, the mean pressure distributions upstream of the interaction to a limited extent and in the interaction region to a large extent are affected by cooling. Thus the boundary-layer displacement thickness on the attached boundary layer approaching the shock is influenced by the cooling. The mean shock Mach number M_1 (Fig. 15), the dynamic pressure levels \bar{p}/q (Fig. 16), the mean interaction length L^*/c (Fig. 17), and C_{nte} (Fig. 14) are all significantly affected by cooling. The interaction length was based on pressure measurements and was calculated by the procedure of Percey¹⁹ and Delery.⁹ For the weak laminar boundary-layer shock interaction on the 14% thick airfoil, with no significant separation, M_1 , \bar{p}/q , L^*/c , and C_{pte} tend to asymptotic values at low T_w/T_{ad} values. For the stronger shock interaction on the 18% thick airfoil values of M_1 , \bar{p}/q , L^*/c , and C_{pte} continued to decrease with reduction in T_w/T_{ad} values and thus do approach a limit. For both interactions, the changes observed are large.

Discussion

Figures 8-17 show that model cooling has large effects on the flow about biconvex airfoils at transonic speeds, which raises interesting questions regarding the simulation of such flows at high Reynolds number.

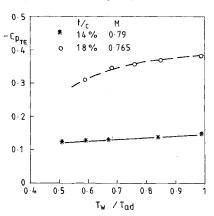


Fig. 14 Variation of pressure near trailing edge with wall temperature ratio.

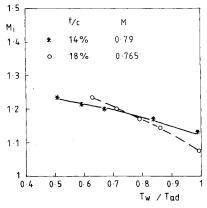


Fig. 15 Variation of shock Mach number with wall temperature ratio.

With regard to the changes in the mean measurements, the changes in pressure distribution could be due to the much thinner laminar shear layers or even a change from a laminar to a turbulent boundary-layer shock wave interaction due to effectively increased surface roughness, pressure gradient, or a combination of these effects.

For an adiabatic flow, reduced trailing-edge pressure divergence would normally be taken to indicate a reduction in the severity of separation effects. If we assume that the separation position is unchanged at the lower values of T_w/T_{ad} (shock position is unchanged), the length of the separation is manifestly unchanged. The change in trailing-edge pressure divergence must be attributed then to a reduction in the thickness of the separation bubble, and this is consistent with what was expected when these tests were planned. ^{10,11} Similar trends, albeit of reduced magnitude, are observed for comparable

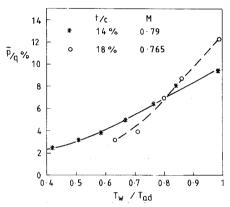


Fig. 16 Variation of peak pressure fluctuations with wall temperature ratio.

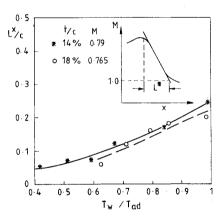


Fig. 17 Variation of interaction length with wall temperature ratio.

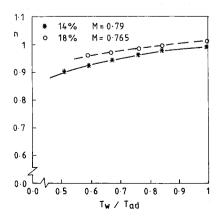


Fig. 18 Variation of nondimensional frequency with wall temperature ratio.

tests with fixed transition (to be published). Hence, taken together, the measurements with both free and fixed transition suggest that flows sensitive to large indirect scale effects can be identified by model cooling as suggested previously. 10,11

With regard to the unsteady measurements, the reduction in the amplitude of periodic pressure fluctuations is consistent with greatly reduced shear layer thicknesses due to model cooling. Figure 18 shows a reduction in n with the decrease in T_w/T_{ad} . It is interesting to consider how the reductions in shear layer thickness reduce the frequency parameter of the oscillations. The frequency of these oscillations is determined by the time taken for disturbances to convect downstream from the shock to the trailing edge (through the shear laver) and also by the time taken for these reflected disturbances to propagate upstream from the trailing edge to the shock (see the discussion of Fig. 13 in Ref. 20). The convection velocity decreases as the parameters fc/u and $f\delta/u$ decrease. Hence the time taken for disturbances to propagate from the shock to the trailing edge will increase, lowering the frequency parameter. Hence cooling the model, which decreases δ , will decrease the convection velocity and lower the frequency parameter, just as observed. It should be noted that also there will be changes in the time taken for disturbances to propagate upstream from the trailing edge to the shock because of the changes in the mean pressure distribution. As the frequency parameter of the periodic flow falls, the aerodynamic resonance moves "off tune" and the amplitude falls. The flow reverts to the normal type of buffet excitation, and hence it is reasonable that the level of random pressure fluctuations should increase. A similar increase in random pressure fluctuations is observed with fixed transition. Hence the decrease in periodic excitation and the increase in random excitation should be observed for high Reynolds number flows with adiabatic wall boundary conditions. This has yet to be confirmed.

Some limitations of these tests must be admitted. So far there has been no opportunity to attempt flow visualization (such as oil flow and high-speed schlieren studies) that were so successful for the adiabatic experiments. 12-14 There has been no opportunity to monitor the state of the boundary layer. Transition has been detected by surface hot films in cryogenic wind tunnels, and hence they should work equally well on cooled models. Despite these limitations, Figs. 8-17 should be of interest to anyone considering the problem of simulating high Reynolds number flows at transonic speeds.

Conclusions

These preliminary tests on biconvex airfoils with laminar boundary-layer interaction show that periodic flows at transonic speeds can be modified strongly by cooling, as expected. The main effects observed as the model is cooled are as follows.

- 1) A marked increase in the local mean Mach number at the mean shock, which remains in much the same position.
- 2) A decrease in trailing-edge mean pressure divergence (indicative of a reduced length scale of the separation normal to the surface).
- 3) A decrease in the amplitude of the periodic pressure fluctuations consistent with a reduced shock amplitude.
- 4) A decrease in the frequency parameter of the periodic shock oscillations, due to the combined effects of increased downstream convection and upstream propagation times.
- 5) Further investigations (to be published) indicate that the preceding conclusions are also valid for an interaction involving a fully turbulent boundary layer.

All of these observations are thought to correspond with what would happen to adiabatic flows at greatly increased Reynolds numbers.

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